Exchange driven growth with a source and sink of particles

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This project extends previous work on exchange-driven growth processes. We consider a system of clusters made up of individual units called monomers. Clusters collide, exchanging monomers back and forth with a probability encoded by an exchange kernel. We set out to investigate the long-term behaviour of the system with both a source and a sink of clusters present, hypothesising the existence of a steady state, where the distribution of cluster sizes eventually reaches a constant – a so-called 'dynamic equilibrium'.

The source was modelled by a constant input of monomers, and the sink was modelled by a choice of either evaporation of one monomer at a time, or desorption of an entire cluster at a time.

A simple consideration of process-counting and some physical assumptions, such as spatial homogeneity and the law of mass action, lead to an infinite system of coupled rate equations; imposing the source and sink corresponds to incorporating extra terms on each ODE. Simulating the system necessitated imposing a cutoff, or maximum cluster size, to make the system finite.

The standard predictor-corrector adaptation to the Forward Euler method to solve the ODE system was implemented in Python. As an analytic proof of the convergence of the scheme seemed intractable, we proceeded to validate the code by ensuring it reproduced some of the analytic results found in existing research papers. The code was also benchmarked to ascertain how long a physical time, and how large a cutoff, could be simulated for a reasonable time.

We then experimented widely by varying the parameters of the model. All results were found to be qualitatively independent of the choice of physical sink. We identified the existence of steady states, rendered exactly by the simulations, and that the system took longer to reach a steady state as the strength ratio of the source to the sink increased. As expected, the steady state had an approximately exponential tail. Monotonic divergence of all moments of the distribution was also observed in the presence of a sufficiently strong source. On the boundary between these cases however, we did not find evidence of a phase transition. Systems with a larger cutoff could exhibit a phase transition for an appropriate change in parameter space.