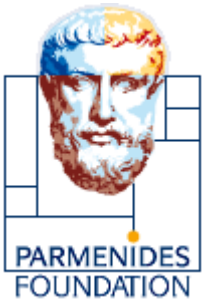

ON TOP AND BEYOND NETWORKS

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Workshop on Approaches to evolution of complex systems held at CSH, Vienna and KLI, Klosterneuburg, November 10-11, 2016

Networks are everywhere – says the blurb of a recent conference.

I would say: interactions are everywhere. And they often are pairwise, and the interacting partners may live on a complex graph.

Today we tend to project networks into everything we see around us.

Sure, it is a great achievement that people (or at least some of them) in fields like e.g. economics realize that there are interactions between the agents (or that there are more agents at all, beyond the lonely representative one) and that the topology of these interactions matter a lot, but I do not think that the story ends there.

In this talk I would like to suggest two directions in which the network picture can be extended. A further direction, which takes into account the effect the nodes can exert on the links and their co-evolution, is the main focus of this Workshop. In the end of this talk I will try to link up to this.

Networks on top of networks

Real world networks have a function, they serve some purpose, they are either spontaneously evolved, or purposefully designed. The efficiency of this function usually depends not only on the local features, but the whole structure, topology, geometry, etc.: there is a *functional* defined over the network.

Furthermore, there may be a success criterion (extremal principle) associated with this functional. For simplicity, I assume that the functional is a real valued scalar and will refer to it as the free energy. Success, optimality, etc. corresponds to the minimum of the functional.

Examples

Transport network

- Functional: # of people, volume of goods transported, GDP

Financial networks

- Functional: stability and/or efficiency of financial intermediation

Interpersonal relationships, relation to institutions

- Functional: social happiness, low transaction costs

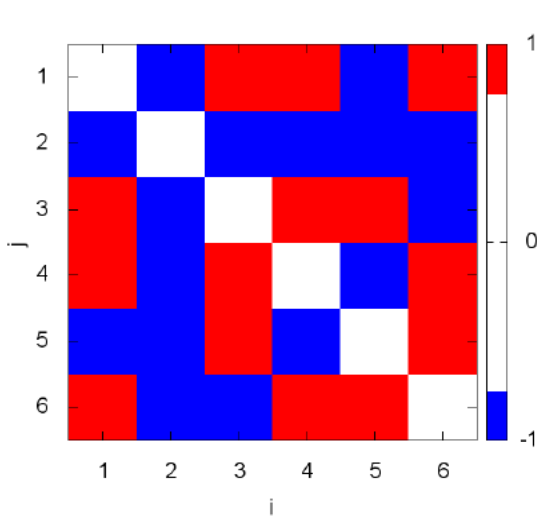
Illustration: Ising spin glass

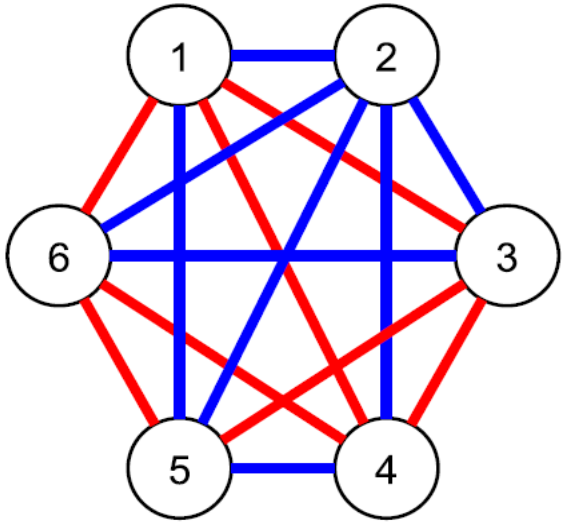
- N agents s_i, i= 1,2,...,N, facing a binary choice: $s_i = \pm 1$ interacting via $J_{i,j} = \pm 1$, and possibly under a bias or external field:

$$E = -\sum_{\langle i,j \rangle} J_{i,j} s_i s_j - \sum_i h_i s_i$$

- Simple model of competition and cooperation.
- For simplicity, consider complete graph.
- Random search for optimum (MC): Moves accepted with prob. 1 if they decrease or leave E unchanged, and accepted with prob. p if E is increased.

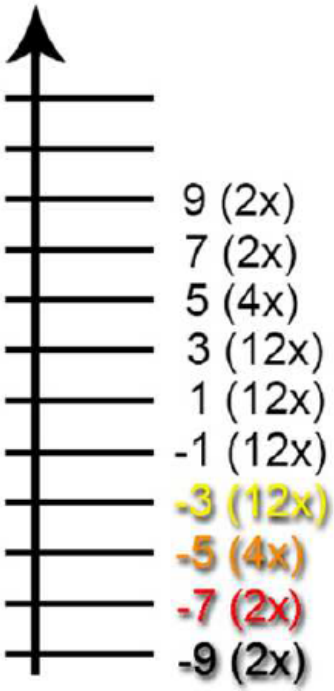
Small sample: red: J=+1, blue: J= -1

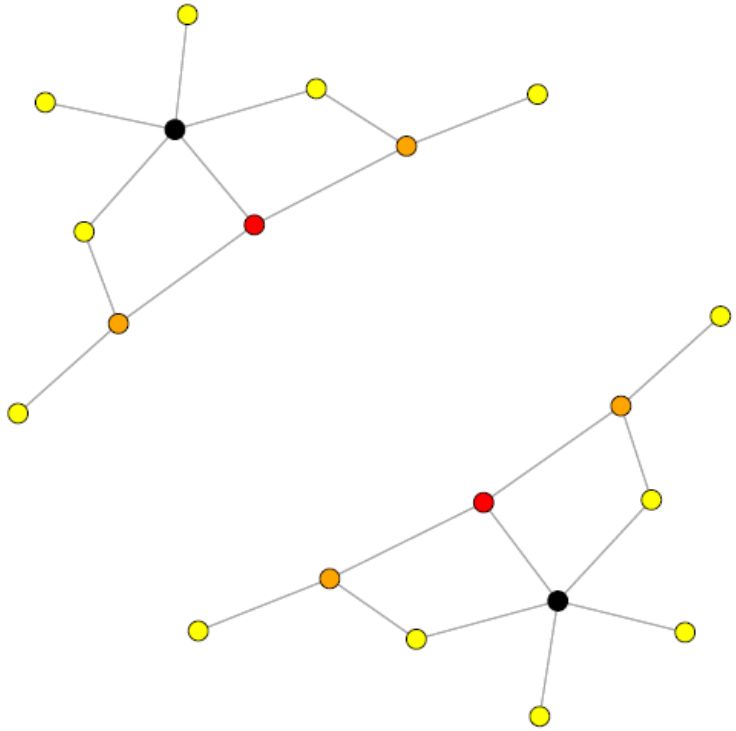




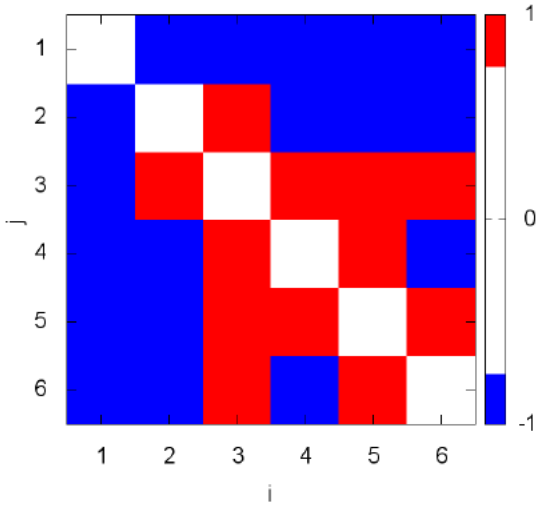
Frustration pushes ground state energy upwards (relative to the every J positive case), increases degeneracy.

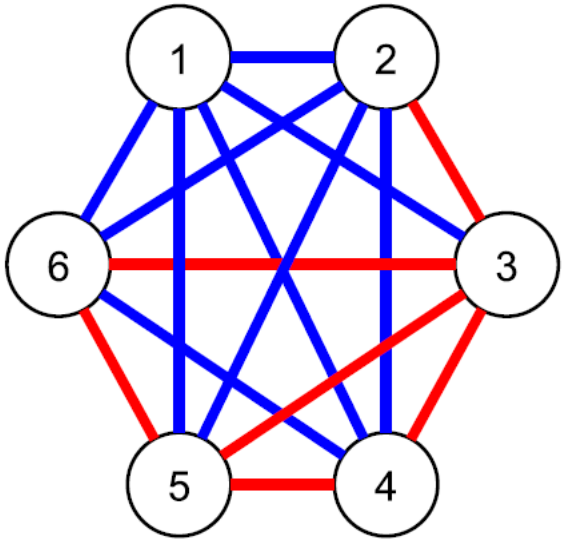
Energy spectrum and network of four lowest lying states



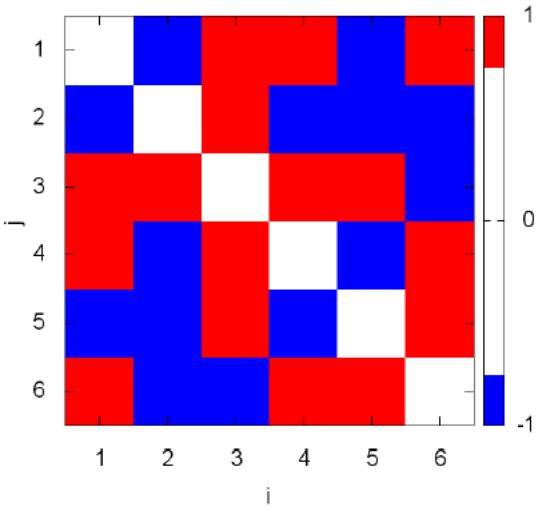


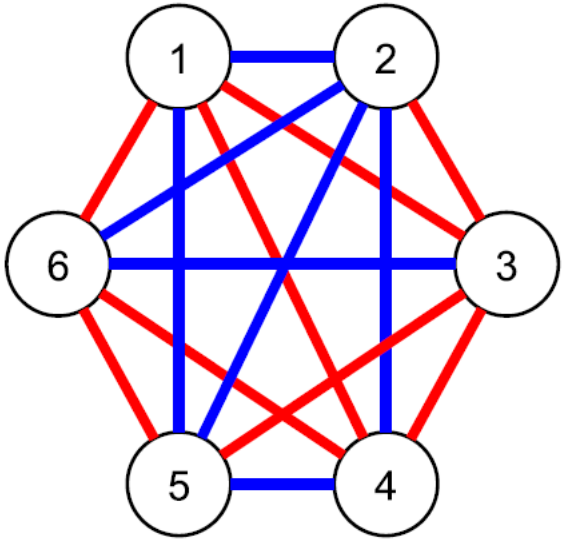
Seemingly different interaction matrix with the same energy landscape (gauge invariance)



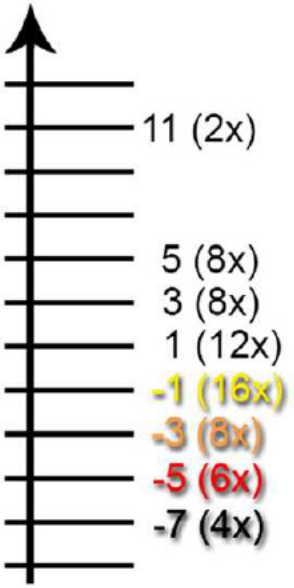


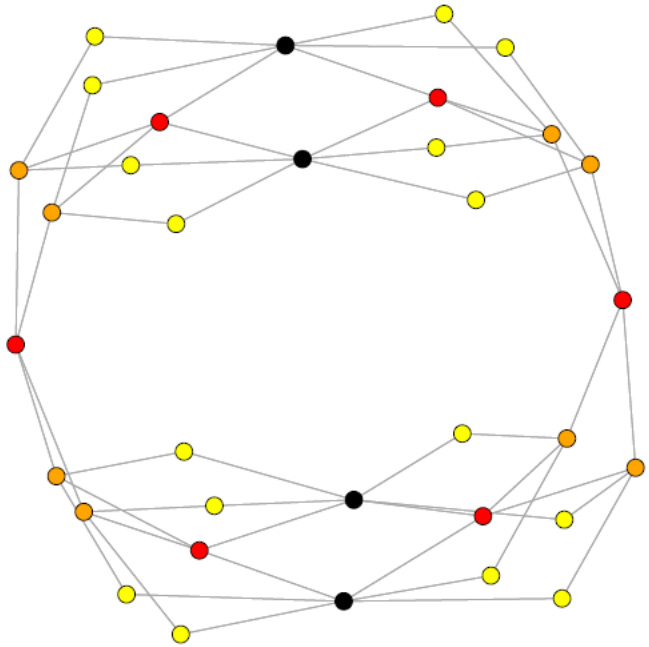
Slightly different interaction matrix with very different landscape



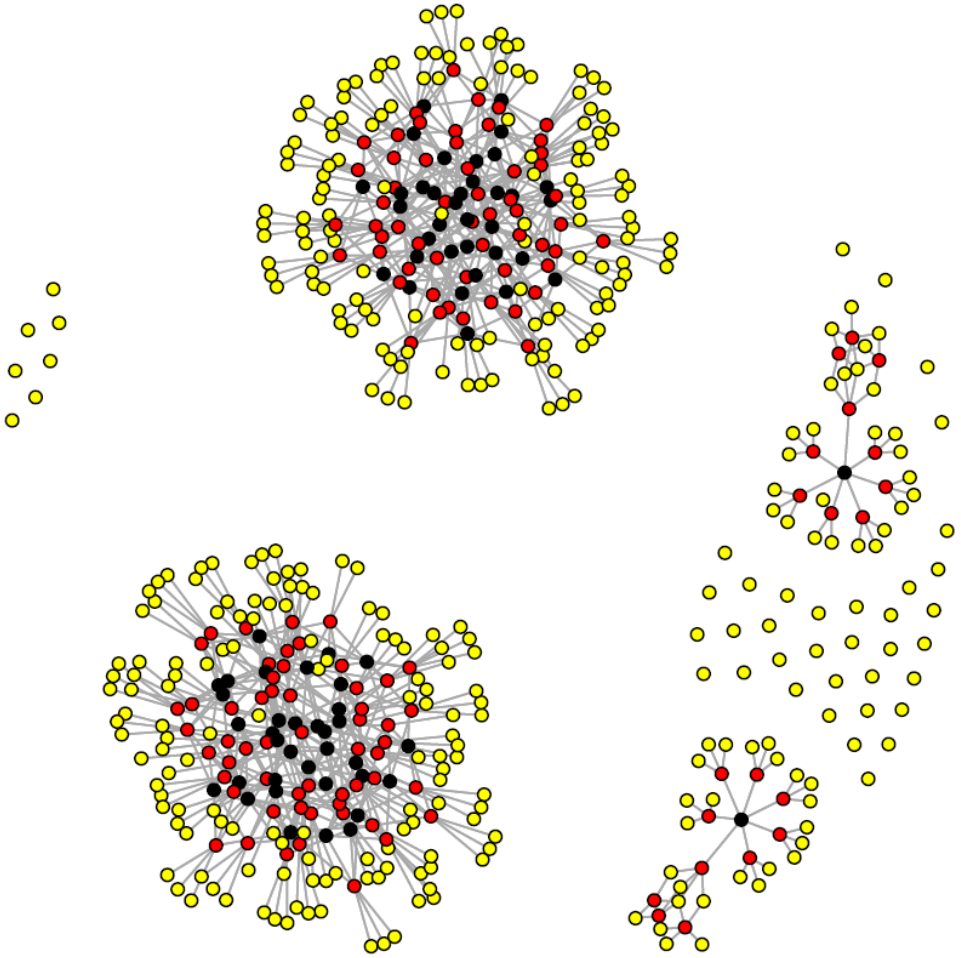


Spectrum and landscape





N=16, lowest 3 energy states that can be reached from each other for small p

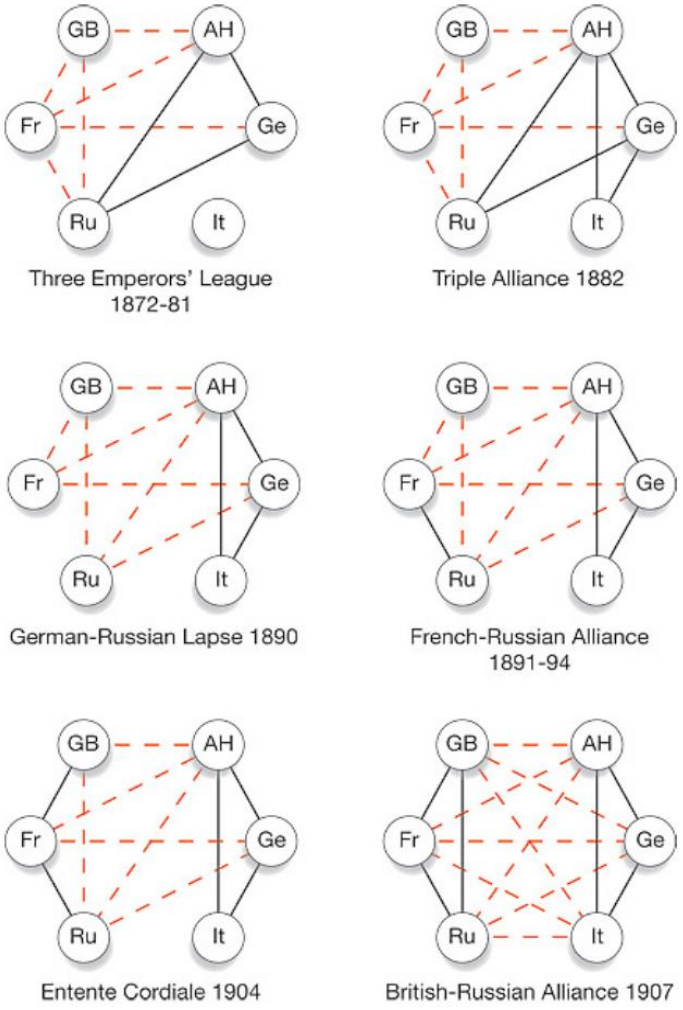


- There is a network of accessible states determined by the functional defined over the underlying network, and the two networks have a very indirect relationship
- Transitions can take place in the phase space landscape without change in the underlying network.
- Slow dynamics, quasi-equilibria, „homeostasis”, punctuated equilibrium.
- Are post-referendum Britain / the US of the 2016 campaign / Orban’s Hungary fundamentally different from their former self?

When the undelying network is changing...

- „Annealed averages”: the underlying network is changing on the same timescale as the agents – leads to boring equilibrium.
- Between annealed and quenched: hardly studied, although the theoretical tools are there.
- Social balance theories: the agents are fixed, but their interactions change so as to reduce overall frustration (the friend of a friend is a friend, and the enemy of an enemy is a friend). Ends up in everyone in the same camp, or two antagonistic camps.

An illustration from history (S. Strogatz)



Beyond networks

In several instances that we try to describe in terms of networks, one can discern some background field that cannot be represented as a sum of binary interactions.

Trust is a major factor of stability of social order or the economy.

Culture, shared values, professional ethics, ideologies, myths, the Zeitgeist, etc. act as external fields on individual agents or their groups, *but also on the interactions* between agents.

Example: banking networks

- In the wake of the 2007-08 crisis a large number of works have been devoted to banking networks and their systemic risk implications. The concept of networks is one of the few new ideas that has penetrated regulation (J.-C. Trichet) and some of the finance literature.
- Magic of words: networks, contagion, cascades evoke the picture of a serial default, forest fire, dominos falling over, etc. What happend on September 15, 2008 was not like this; Bernanke, Paulson and Geithner desperately tried to find a buyer for Lehman, but no other bank wanted to buy it. The credit market seized up in two days.
- It was a total collapse of trust.

The week of the crisis

MONDAY

Falling Giants

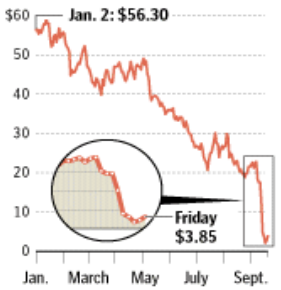
- Lehman Brothers files for bankruptcy protection.
- Bank of America agrees to buy Merrill Lynch.
- The Federal Reserve elects not to raise interest rates.
- Oil drops below $100 a barrel.

TUESDAY

Big Bailout

- The Fed lends insurance giant American International Group (AIG) $85 billion in exchange for nearly 80 percent of its stock.
- Lawmakers contemplate other dramatic interventions.

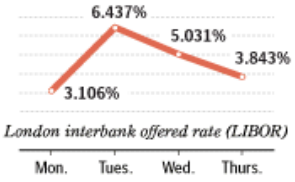
AIG stock price, daily close



WEDNESDAY

Lending Locks Up

- As banks abruptly stop lending to each other, cash becomes scarce, driving up the cost of capital.
- Washington Mutual puts itself up for sale.
- Morgan Stanley and Goldman Sachs shares drop 24 and 14 percent respectively.
- Morgan Stanley and Wachovia enter merger talks.
- The price of gold soars more than 8 percent.
- The SEC adopts new rules against abusive "naked short-selling."



London interbank offered rate (LIBOR)

THURSDAY

Global Markets Unite

- The Fed and central banks in Europe, Japan and Canada team up to inject as much as $180 billion into global markets to ease the cash crunch.

Global Lenders

FRIDAY

Addressing the "Root" Problem

- President Bush announces plan to buy troubled assets from financial firms.
- Money market funds offered protection.
- Fannie Mae and Freddie Mac to increase purchases of mortgage-backed securities.
- SEC temporarily bans short-selling in shares of nearly 800 financial institutions.

Banking networks

- Banks are connected by overlapping asset portfolios, mutual exposures, credit-debt relationships, etc.
- But also: by common culture, common ideology, common business school education, common pubs, golf clubs, etc., employees subject to same toxic atmosphere and overburden, rollercoaster of panic and exuberance.
- The former can be modelled by networks, the latter cannot.
- How should we model such a turbulent ecology?

The kind of model I can imagine

- Update interactions by an amount depending on the current state (measured in terms of energy, frustration, magnetization...) of the system, shifting local fields and some of the couplings upwards (more cooperation), or downwards (more competition) proportionally, let the system relax, then repeat.
- A timid step in this direction: repeated strategic voting, where agents have a target outcome and modify their next vote such as to pull outcome closer to their target. Leads to fixed point or 2-cycle.

THANK YOU!