On the effect of resetting in random walks on graphs

Dr. Colm Connaughton   Santiago Guzmán Pro   Dr. Fernando Metz

London Mathematical Laboratory

LML Summer School 2018
London, England

On the effect of stochastic resetting in diffusion processes

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Overview

Simple diffusion

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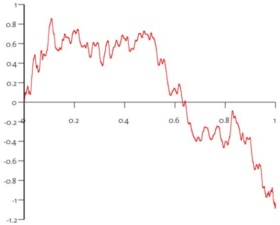
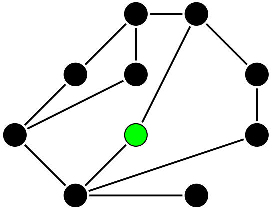
A model of a particle moving randomly in space

- Memoryless
- Time homogenous
- *Space homogenous*



Simple diffusion in the real line



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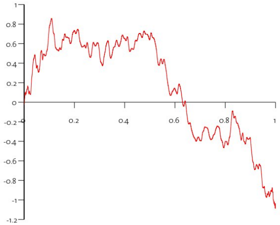

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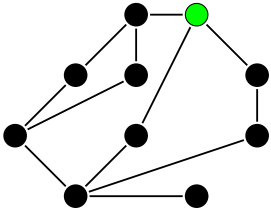

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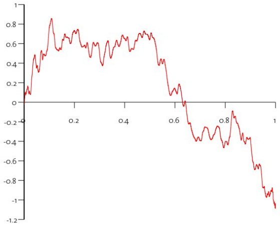

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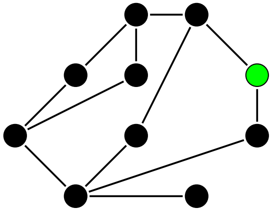

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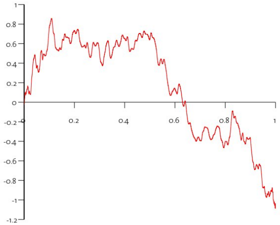

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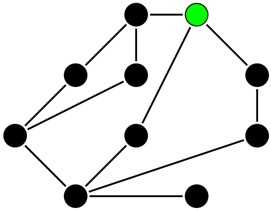

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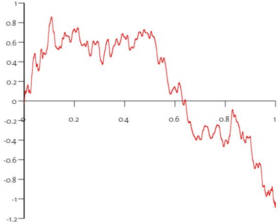

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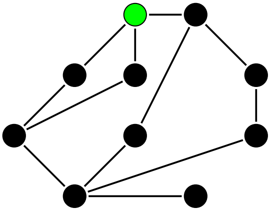

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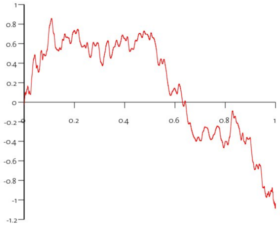

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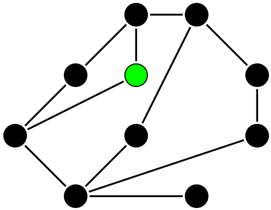

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Stochastic Resetting

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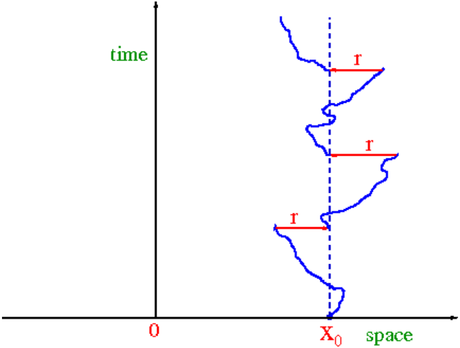

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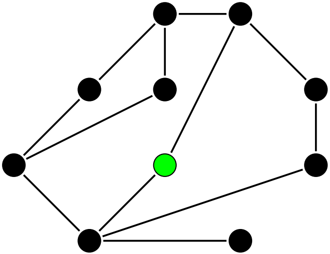

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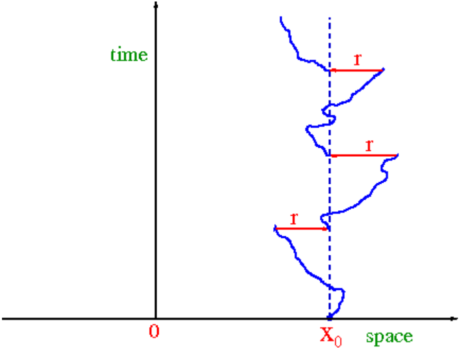

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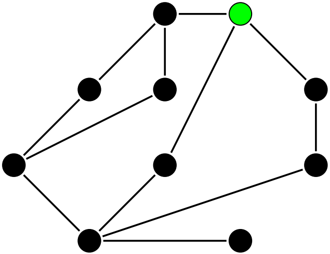

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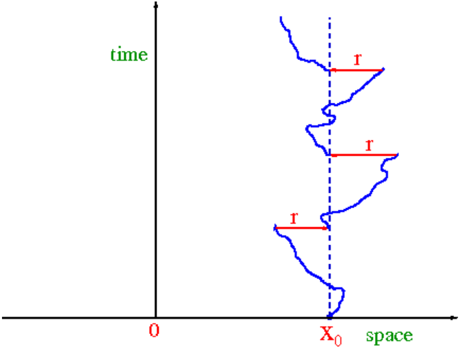

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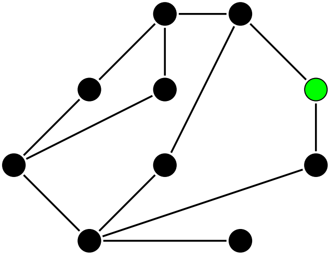

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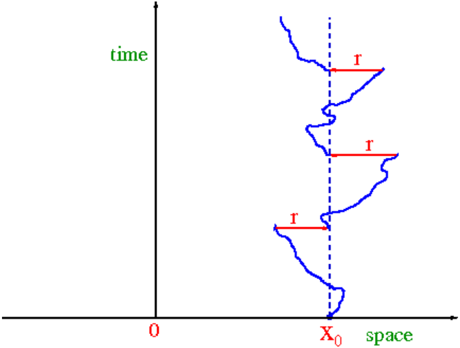

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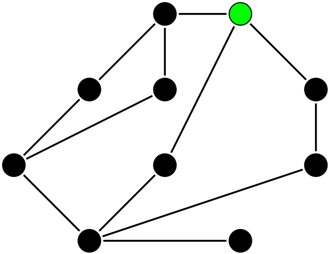

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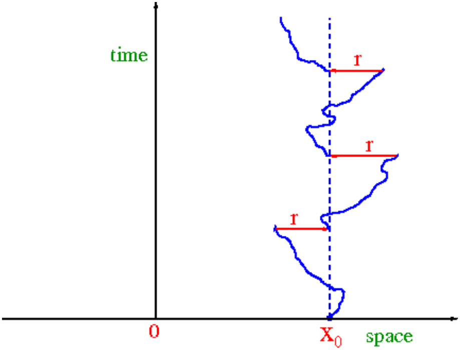

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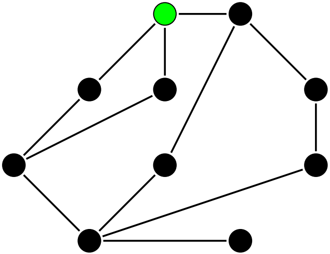

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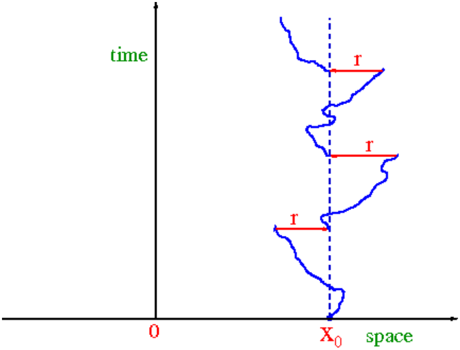

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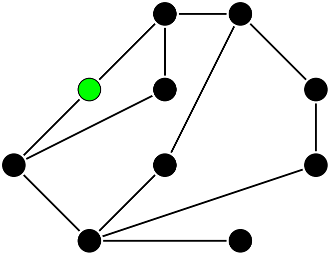

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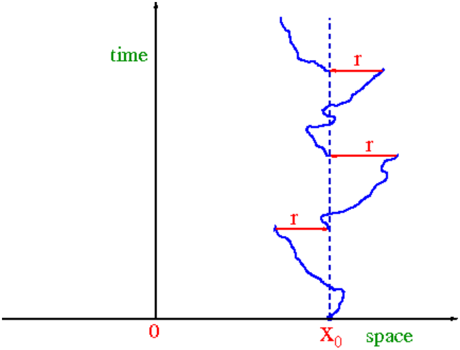

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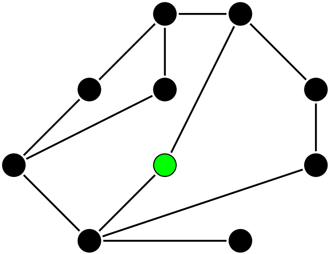

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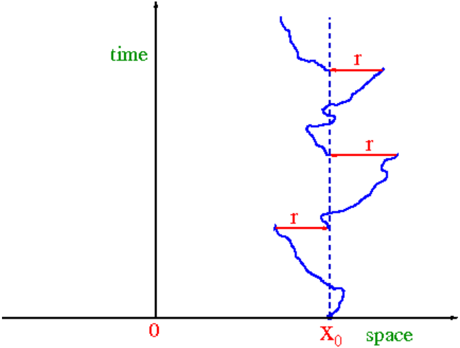

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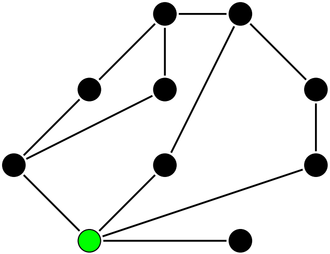

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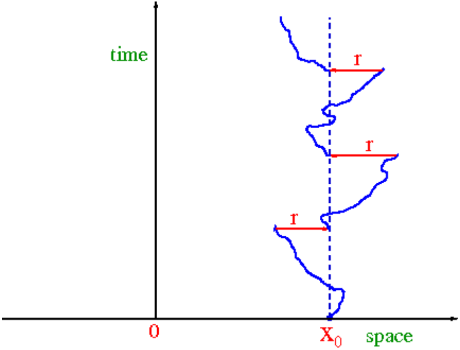

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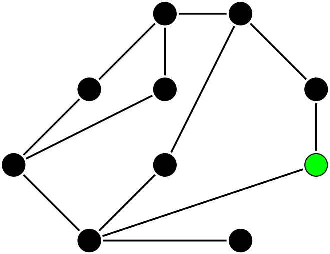

Simple random walk on a graph with resetting

Main interest

How long it will take a particle to hit a given target?

- Define the RV $H_x = \inf\{t \in \mathbb{R} | X_t = x\}$
- Define the Mean First Passage Time as $T(x) := \mathbb{E}(H_x)$
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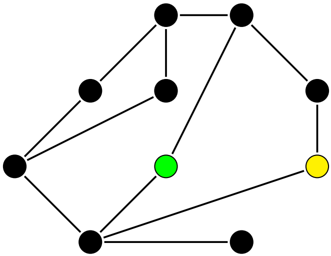

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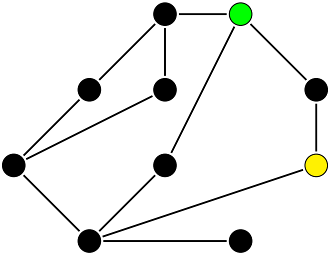

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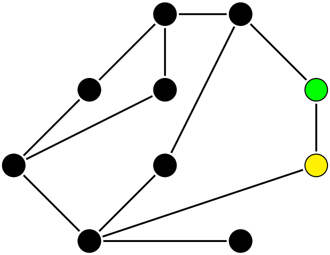

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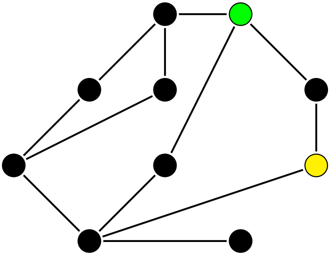

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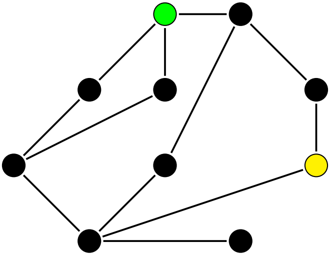

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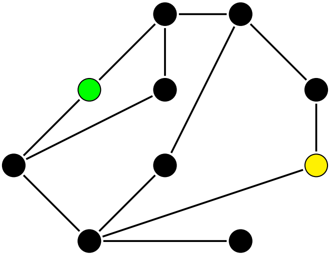

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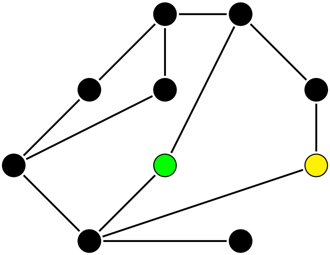

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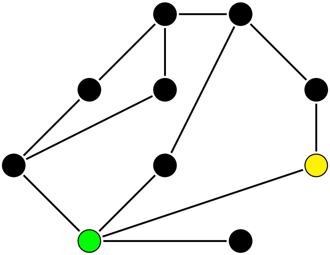

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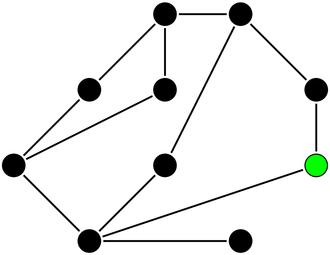

Simple random walk on a graph with resetting

Martin R. Evans and Satya N. Majumdar

Master equation

$$\frac{\delta p(x,t|x_o)}{\delta t} = D\frac{\delta^2 p(x,t|x_o)}{\delta x^2} - rp(x,t|x_o) + r\delta(x - x_o)$$

Mean First Passage Time

- In the absence of resetting $T(x) = \infty$

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- Moreover, $\lim_{r\to\infty} T_r(x) = \infty$
- Hence, optimal resetting rate for every point x.

Simple diffusion with stochastic resetting in $\mathbb{R}^n$...

- Stationary distribution
- Finite MFPTs
- Optimal resetting rates for **all** points

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Our guideline

The behavior of $T_r(x)$ as a function of r in other spaces.

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Our guideline

The behavior of $T_r(x)$ as a function of r in other spaces.

- Discrete spaces (graphs)
- Compact spaces

Simple diffusion with stochastic resetting in $\mathbb{R}/\mathbb{Z}$

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$$\frac{\delta p(x,t|x_o)}{\delta t} = D\frac{\delta^2 p(x,t|x_o)}{\delta x^2} - rp(x,t|x_o) + r\delta(x-x_o)$$

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Calculating the Mean First Passage Time

- Thoroughly study Evans and Majumdar procedure for finding $T_r(x)$

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Calculating the Mean First Passage Time

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- Follow a similar procedure to theirs.

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$$T(r,x) = \frac{cosh(\frac{\alpha_o}{2})sech(\frac{\alpha_o}{2}(1-2x))-1}{r}$$

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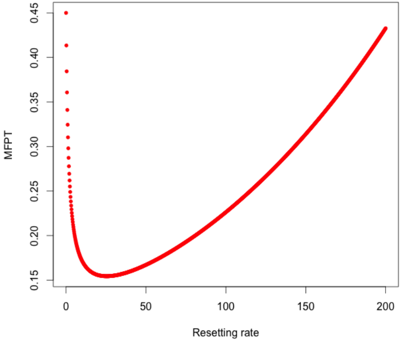
MFPT for a fixed point as a function of rate.
$D = 0.1$, $[x] = [0.1]$



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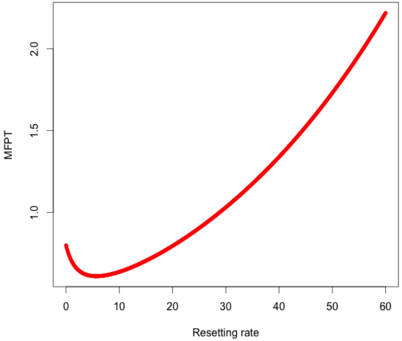
MFPT for a fixed point as a function of rate.
$D = 0.1$, $[x] = [0.2]$



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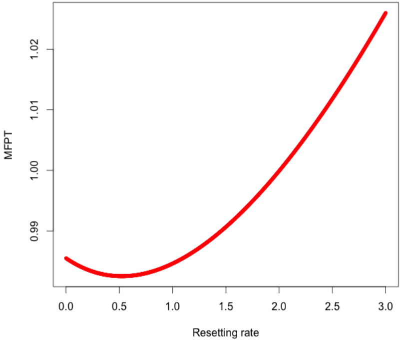
MFPT for a fixed point as a function of rate.
$D = 0.1$, $[x] = [0.27]$



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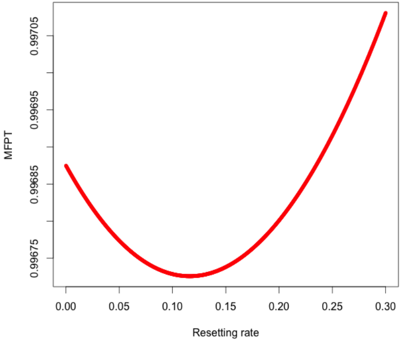
MFPT for a fixed point as a function of rate.
$D = 0.1$, $[x] = [0.275]$



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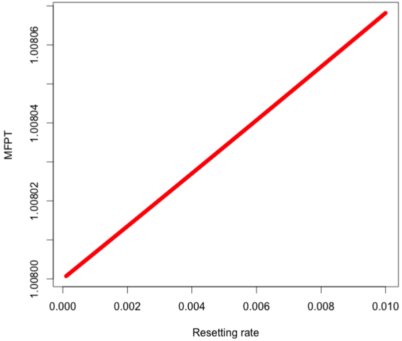
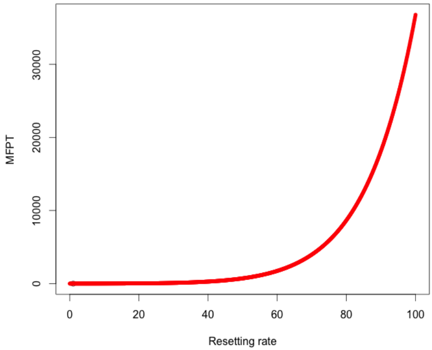
MFPT for a fixed point as a function of rate.
$D = 0.1$, $[x] = [0.28]$



Simple diffusion with stochastic resetting in $\mathbb{R}/\mathbb{Z}$

Mean First Passage Time $[x] = [0.5]$

$$T(r, 0.5) = \frac{cosh(\frac{\alpha_o}{2}) - 1}{r} = \frac{1}{8D} + \sum_{n=2}^{\infty} \frac{r^{n-1}}{D^n 2n! 2^{2n}}$$



Simple diffusion with stochastic resetting in $\mathbb{R}/\mathbb{Z}$

Mean First Passage Time power series

$$T(r,x) = \frac{x(1-x)}{2D} + \frac{x(x-1)(10x-5-\sqrt{5})(10x-5+\sqrt{5})}{24D^2}r + O(r^2)$$

Simple diffusion with stochastic resetting in $\mathbb{R}/\mathbb{Z}$

Mean First Passage Time power series

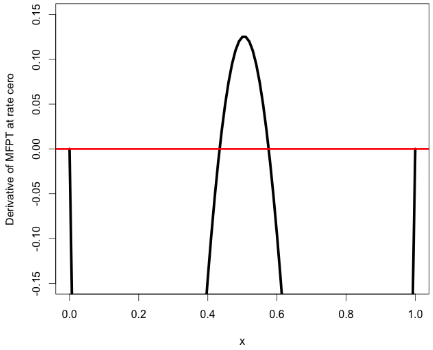
$$T(r,x) = \frac{x(1-x)}{2D} + \frac{x(x-1)(10x-5-\sqrt{5})(10x-5+\sqrt{5})}{24D^2} r + O(r^2)$$

$$\frac{\delta T(r,x)}{\delta r} = \frac{x(x-1)(10x-5-\sqrt{5})(10x-5+\sqrt{5})}{24D^2} + O(r)$$

Simple diffusion with stochastic resetting in $\mathbb{R}/\mathbb{Z}$

First derivative of $T(r,x)$ at $r=0$

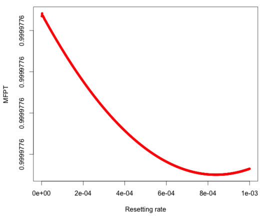
$$\frac{\delta T(r,x)}{\delta r}(0,x) = \frac{x(x-1)(10x-5-\sqrt{5})(10x-5+\sqrt{5})}{24D^2}$$



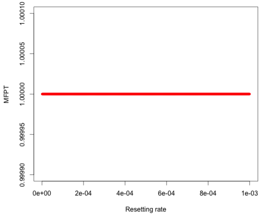
Simple diffusion with stochastic resetting in $\mathbb{R}/\mathbb{Z}$

Main Result

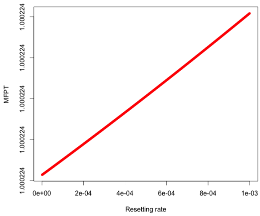
For $x \in (0,1)$, $T(r,x)$ is monotonic if and only if $x \in [0.5-\tau, 0.5+\tau]$, where $\tau = \frac{\sqrt{5}}{10}$.



$x = 0.5 - \tau - 10^{-3}$



$x = 0.5 - \tau$

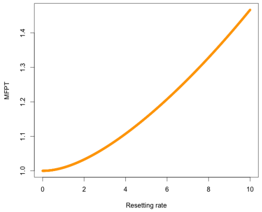


$x = 0.5 - \tau + 10^{-3}$

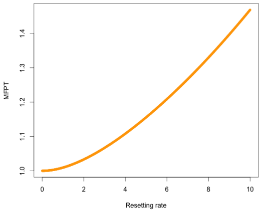
Simple diffusion with stochastic resetting in $\mathbb{R}/\mathbb{Z}$

Main Result

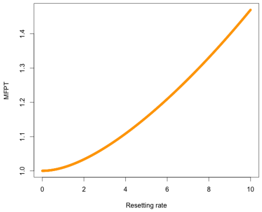
For $x \in (0,1)$, $T(r,x)$ is monotonic if and only if $x \in [0.5-\tau, 0.5+\tau]$, where $\tau = \frac{\sqrt{5}}{10}$.



$x = 0.5 - \tau - 10^{-3}$



$x = 0.5 - \tau$



$x = 0.5 - \tau + 10^{-3}$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OBRIGADO
TEŞEKKÜR
GRACIAS
DHANYAVĀD
KÖSZÖNÖM
KIITOS
TAK
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MERCI
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GRÀCIES
THANK YOU
SHUKRAN
XIÈXIÈ
DĚKUJI
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GRAZIE
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