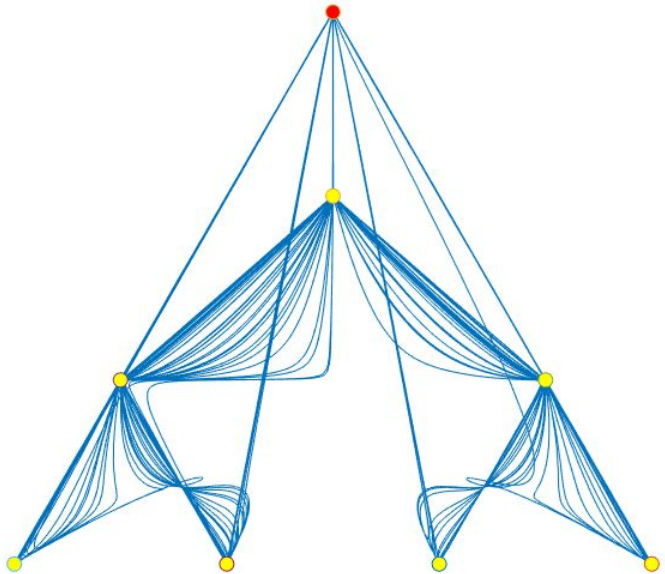
From microscopic to macroscopic noise: the dynamics of transitions around noisy networks



George Wynne
Supervisors: Rosemary Harris & Claire Postlethwaite

Introduction

- We study of heteroclinic networks which are collections of heteroclinic cycles, essentially just the phase space of some ODE.

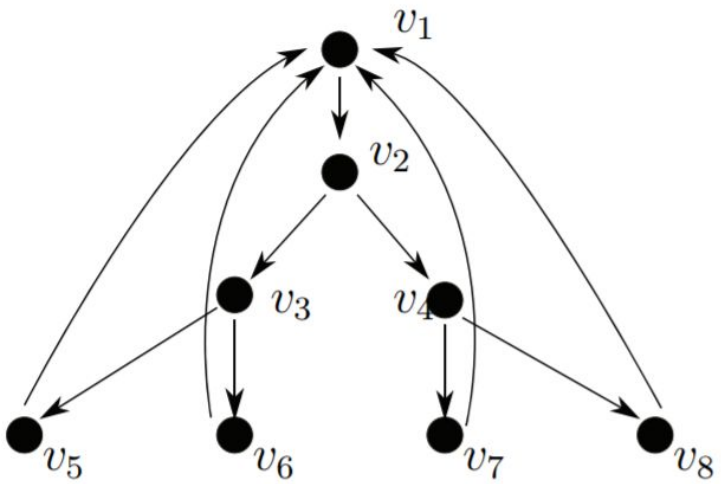
- We then add noise to the ODE and see what happen.

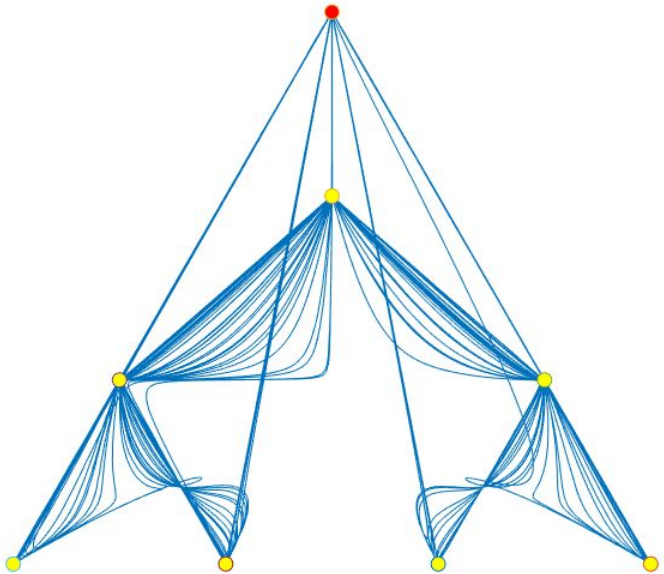
- The micro effects are the movement near equilibria, the macro is the sequence of equilibria visited.

Example trajectories

- By changing parameters in the below equation we can ‘realise’ a large class of graphs

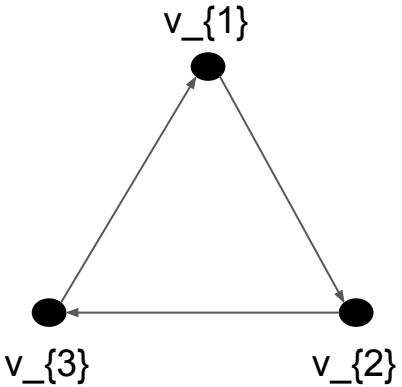
$$\frac{dx_j}{dt} = x_j(1 - |x|^2 + \sum_{i=1}^{n} a_{ij}\, x_i^2) + \varepsilon w_j(t)$$

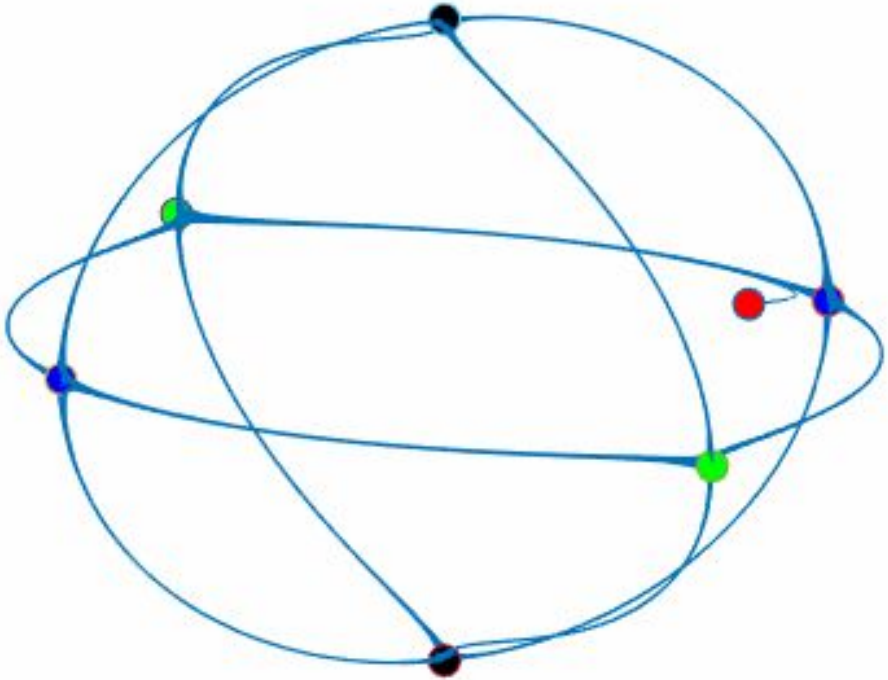




Example trajectories

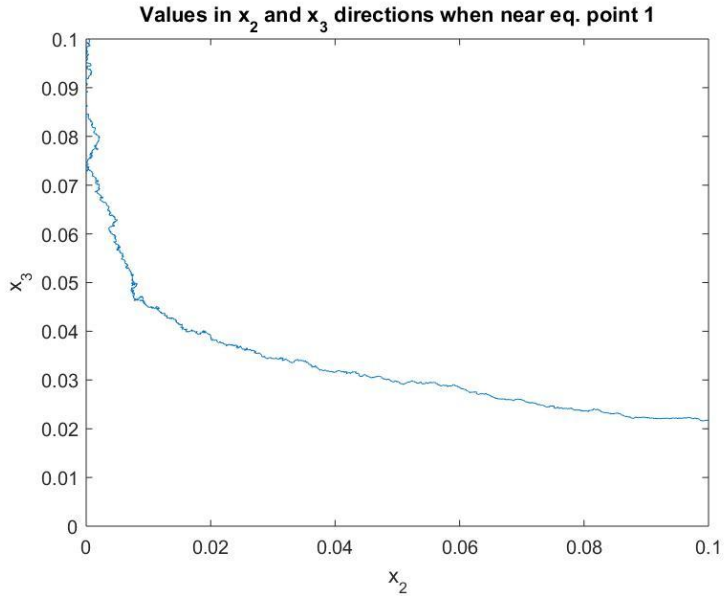
- Blue points = (+- 1,0,0), Green points = (0,+-1,0), Black points = (0,0,+-1)
- Order of equilibria is 1,2,3,1,2,3,1

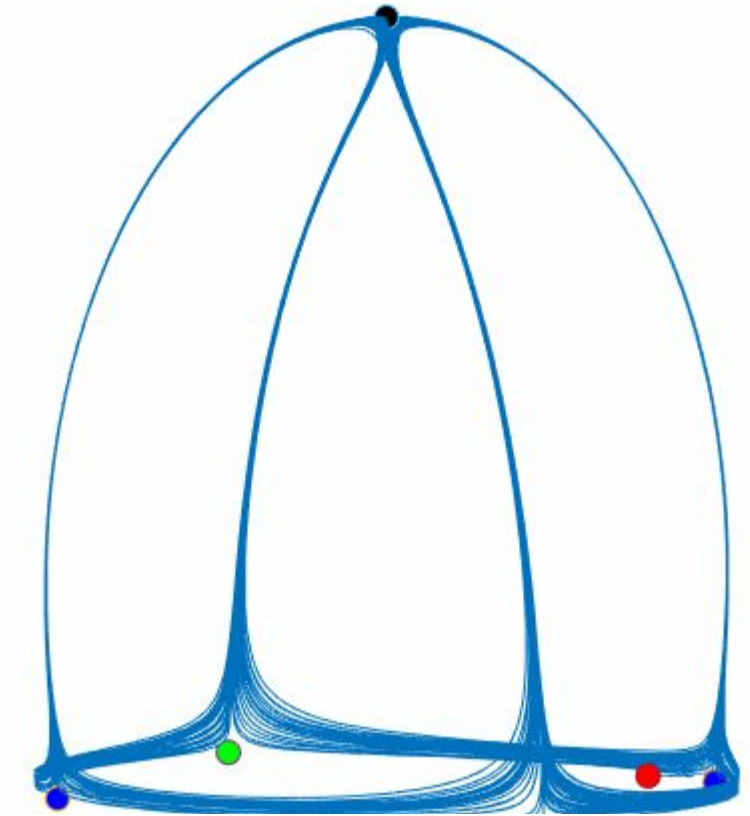




Local effects from linearising equations

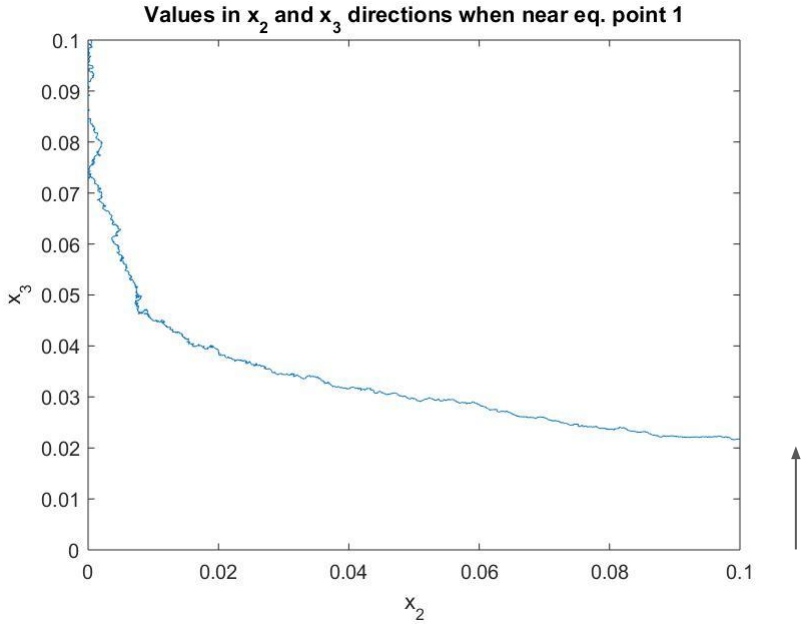
- ‘Lift-off’ occurs when the contraction from the previous equilibrium point is weaker than expansion to the next equilibrium point.

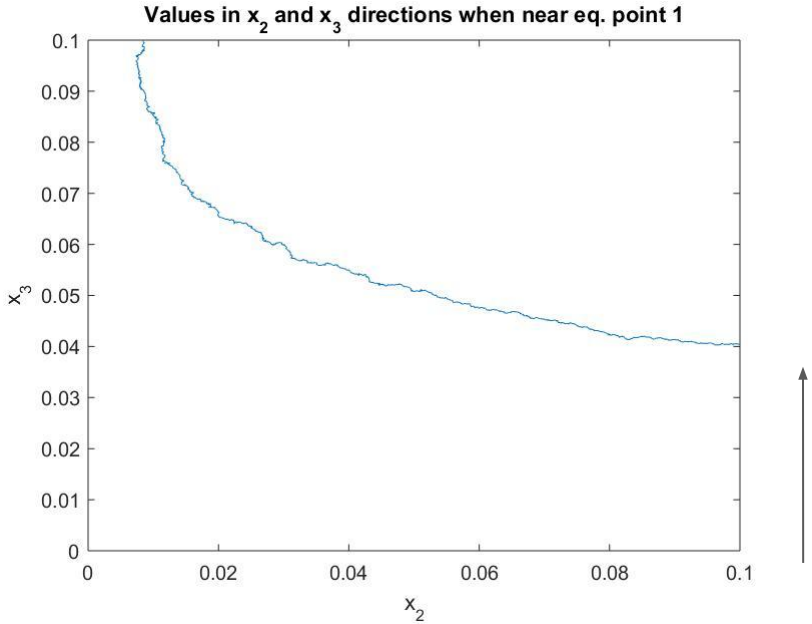




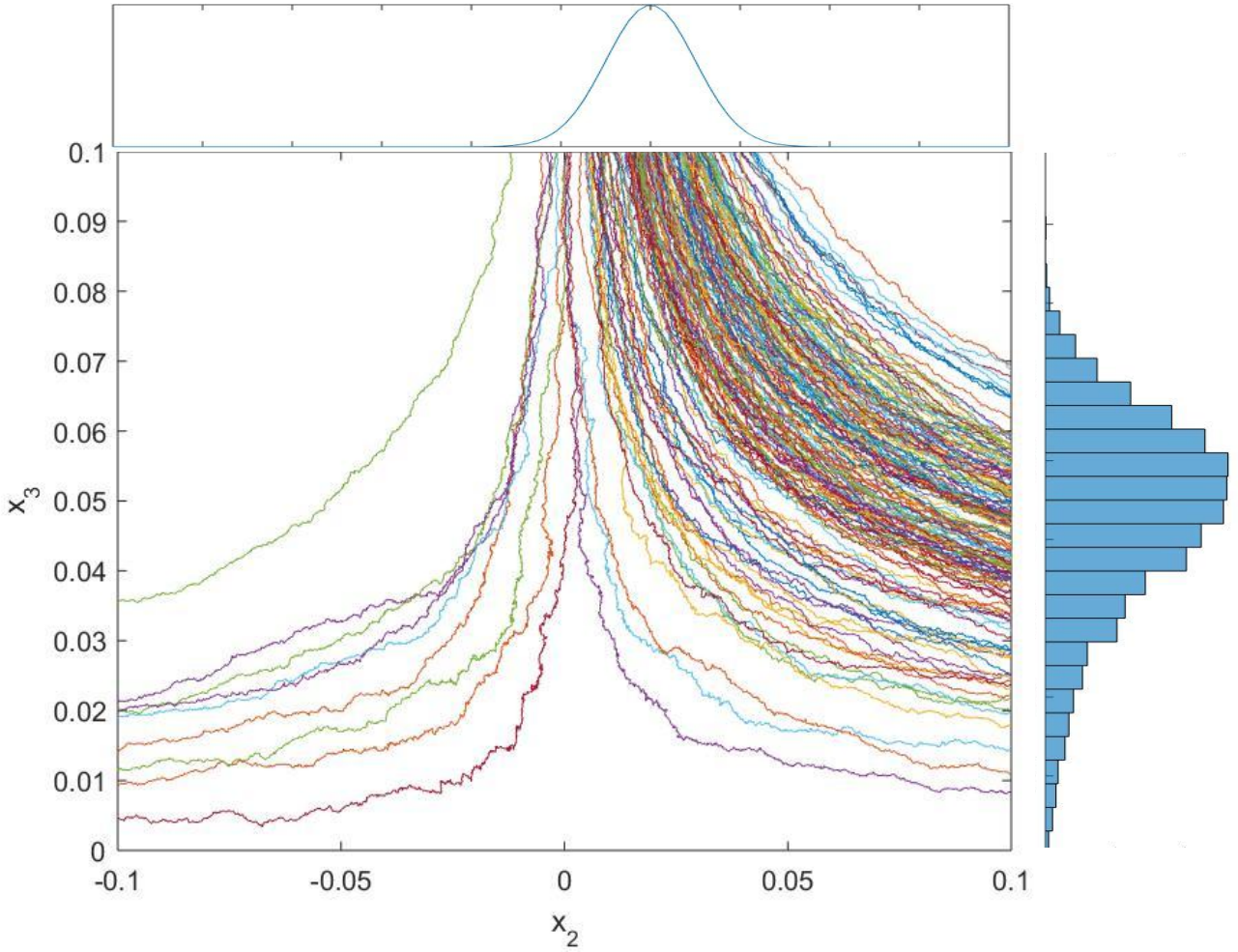
OU Simulations

- Previous analysis of this lift of assumed the initial value of the outgoing direction was zero. I investigated the effects of it starting at a non-zero value. This is reasonable to investigate since lift-off could have been propagated through the network





Input distribution is positive mean Gaussian, histogram is obtain from simulations of OU process

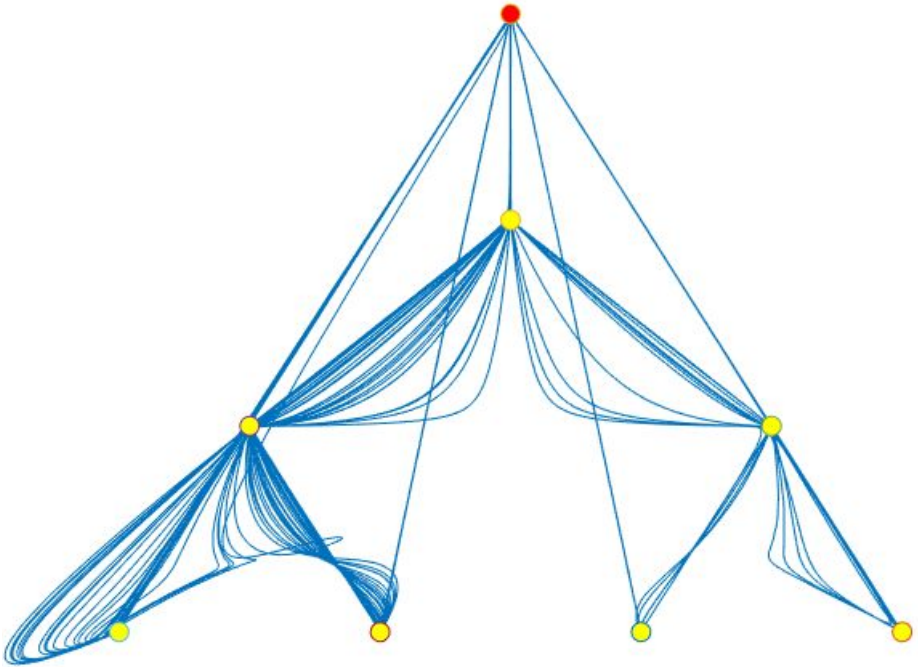


Macro effects

- The micro effects can cause different paths of the network to be explored.

- This raises the question of memory effects in the network

- Compounding lift-off can get complicated



What was found

- Closed form formula for neighbourhood hitting time of exit direction
- Integrated this against solution of OU process to get distribution of lift-offs at the time when the particle leaves the neighbourhood
- Obtained approximations of its mean & derived equations for the mode of the hitting time distribution.
- Obtained noise scaling result of mean lift-off in the context of previous lift-off having occurred in the network.

What still needs to be investigated

- Results applying these lift-off effects in more complicated networks with multiple input and output directions at equilibria
- Understanding analytic properties of lift-off distribution
- Obtaining better approximations for mean lift-off in terms of the noise parameter
- Simulations to check whether Gaussian lift-off distribution is valid in more complicated networks

Thank you for listening!