The Sublime in Maths and Science

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Motivation

- Sublime has meaning in both mathematics and philosophy
- Are these descriptions referring to the same thing?

Key Aims

- Understand the concept of a sublime experience in terms of philosophy
- Establish compatibility of mathematics with this concept
- Seek examples to corroborate findings

History of the Sublime

- References date back to 1st century AD
- Found in literature, art, science and is studied in philosophy
- Philosophy tries to define a sublime experience
- Pleasure stemming from displeasure: awe and fear

Immanuel Kant (1724-1804)

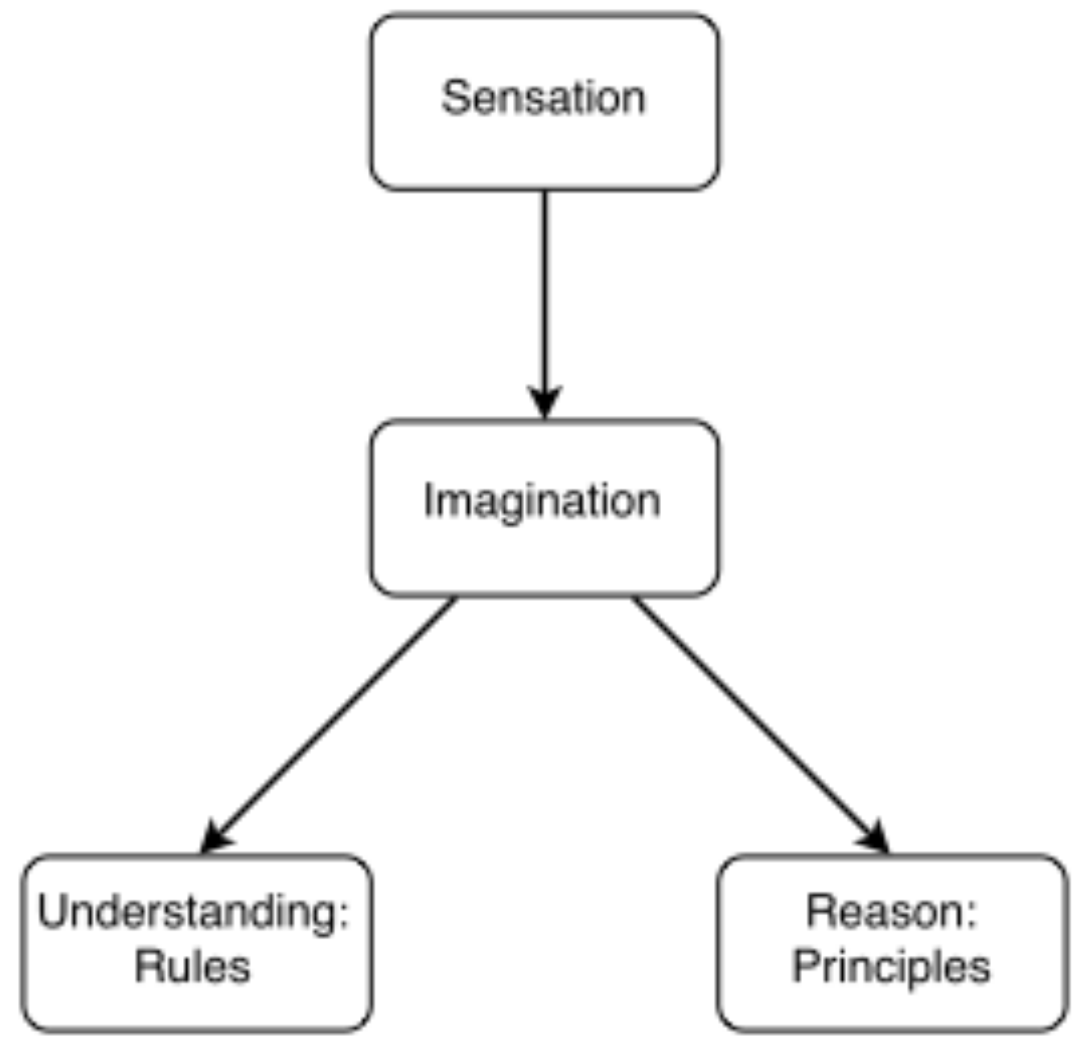
- Analysed the sublime in 1760 and 1794
- Logical and systematic

Kant’s Aesthetics

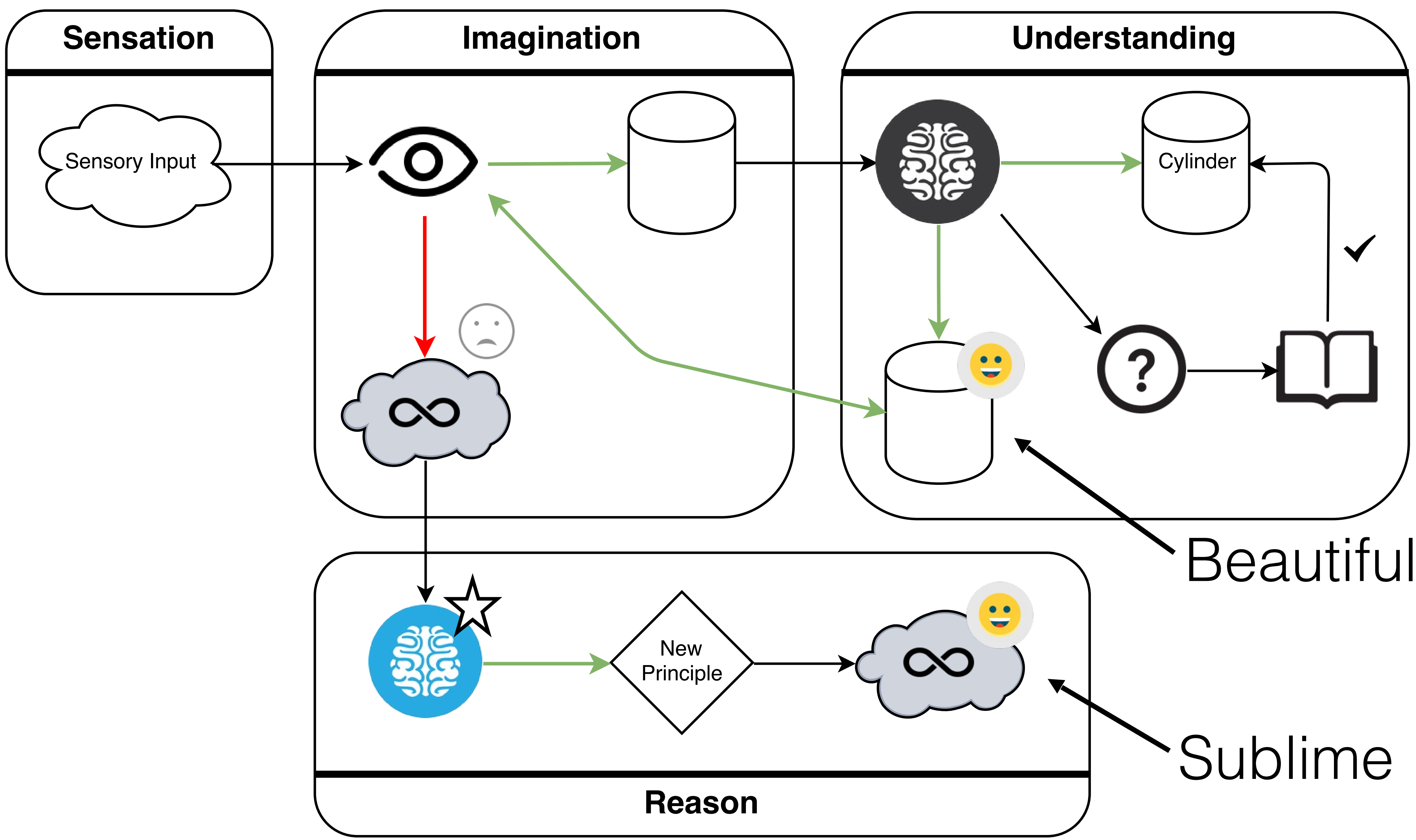
- Judgements without referral to a concept or prior notion
- Beauty: purely pleasurable
- Dynamical Sublime: fearful of the power of nature, whilst being safe
- Mathematical Sublime: unable to comprehend magnitude, need to access a ‘super sensible faculty’ - Reason.

Process of Judgement:

- **Imagination** ‘gives form’ to the sensory data, also called *perception*
- **Understanding** applies a rule if imagination succeeds
- **Reason** deals in principles, universals, totality



Process of Judgement:



Mathematical Compatibility

- Non-aesthetic: refer to purpose of object during judgement
- Mathematical objects unable to be aesthetically judged
- Intellectual pleasure instead

Beauty in Proofs - Angela Breitenbach (2013)

- Breitenbach argues Kant allows for beauty in mathematics
- Beauty is <u>not</u> in mathematical objects, or their properties
- BUT demonstration or proof of a property <u>can</u> be beautiful i.e. the judgement is aesthetic

Two types of Infinity

- Potential: unbounded, limitless, accepted
- Actual: complete, whole, controversial

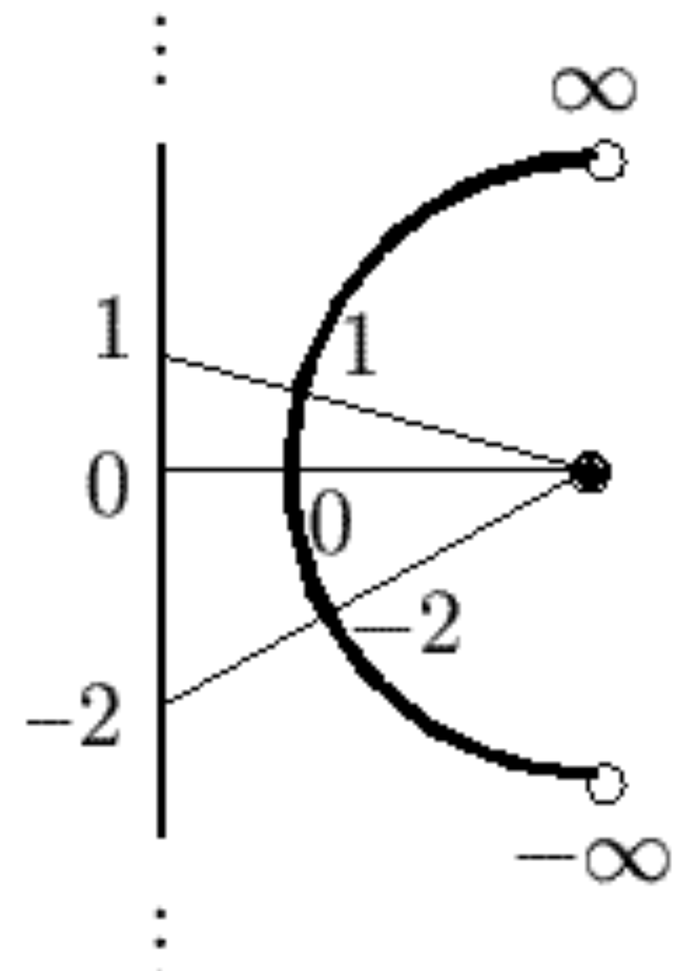
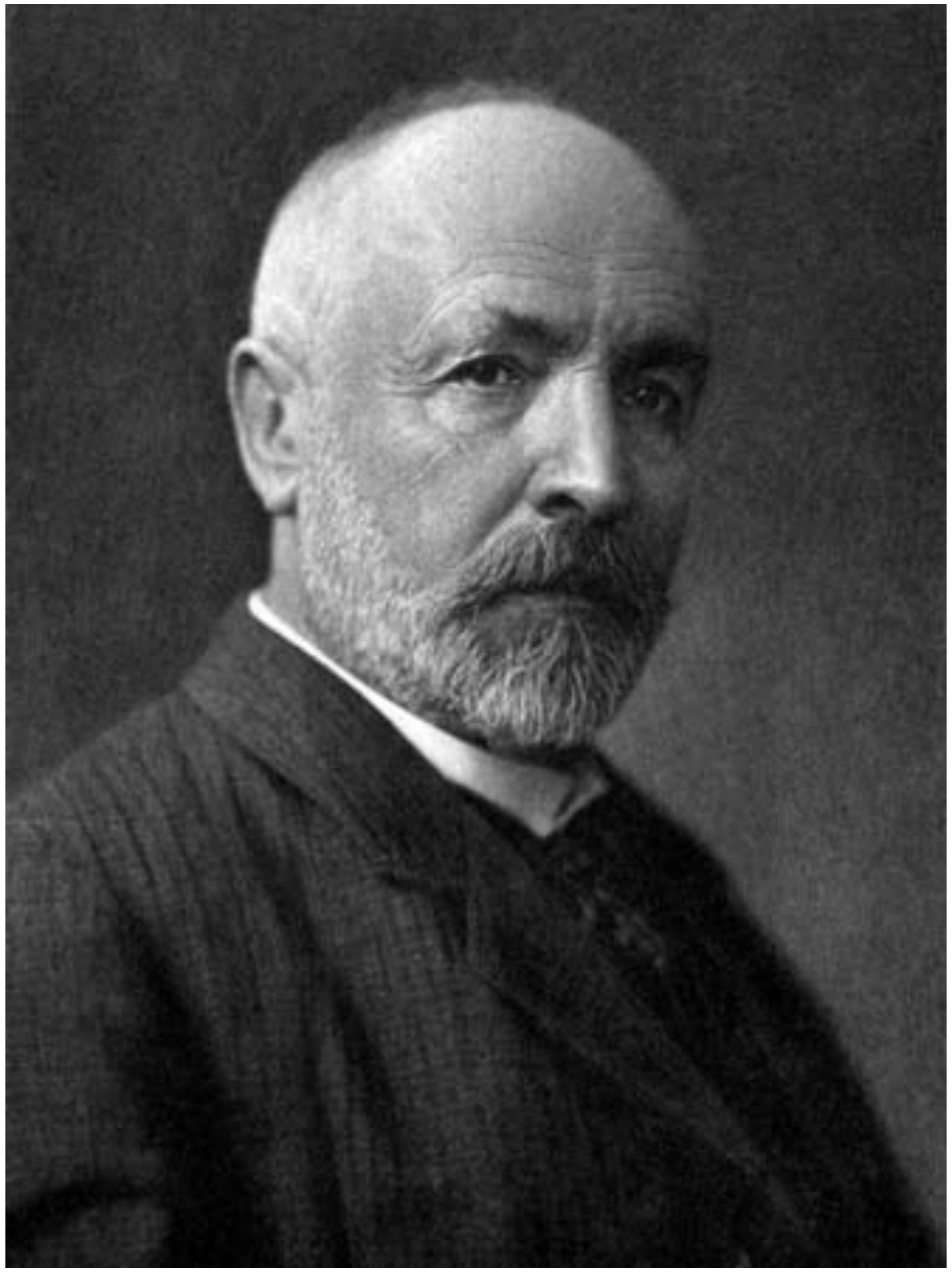


Image from: “Potential versus Completed Infinity: its history and controversy” - E. Schechter

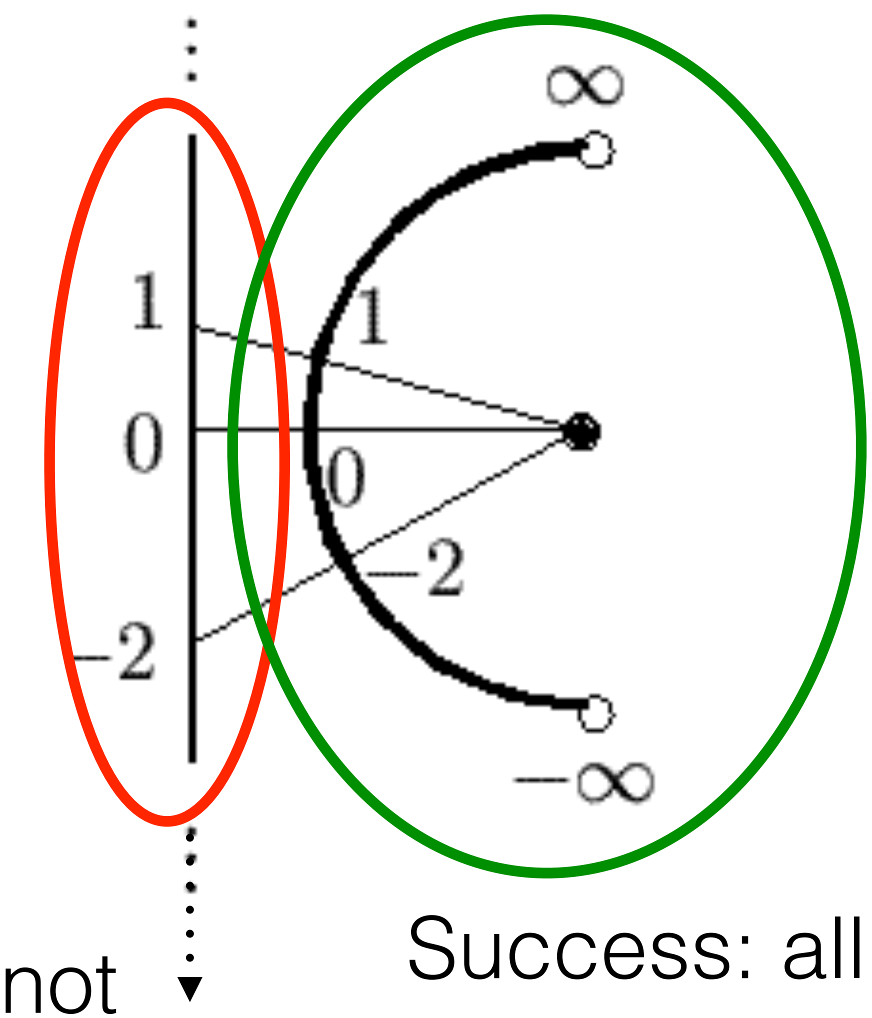
Using Infinities: Cantor

- Potential infinities relate to imagination failing to delimit
- Actual infinities relate to reason: condense an ungraspable idea into a neat package
- Georg Cantor's diagonalization argument: defending actual infinities



Criteria:

- **Aesthetic:** avoiding concepts?
- **Imagination:** spontaneous?
- **Purposiveness:** imagination succeeds or fails in delimiting?



Counting

- Countability is equivalent to being able to write a numbered list of all the elements in a set

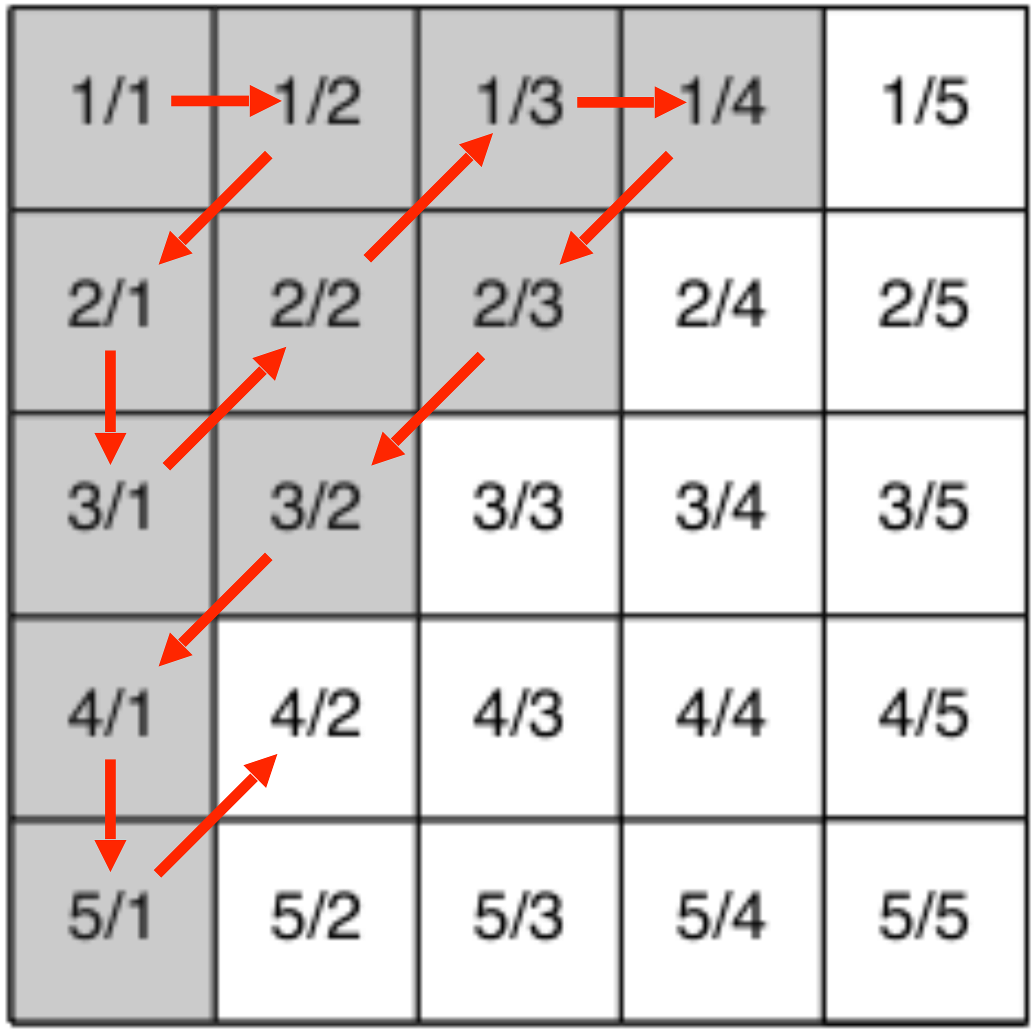
The Rational Numbers

- Arrange rational numbers in a grid
- Horizontal counting: will run out of natural numbers before reaching second row

1/1	1/2	1/3	1/4	1/5
2/1	2/2	2/3	2/4	2/5
3/1	3/2	3/3	3/4	3/5
4/1	4/2	4/3	4/4	4/5
5/1	5/2	5/3	5/4	5/5

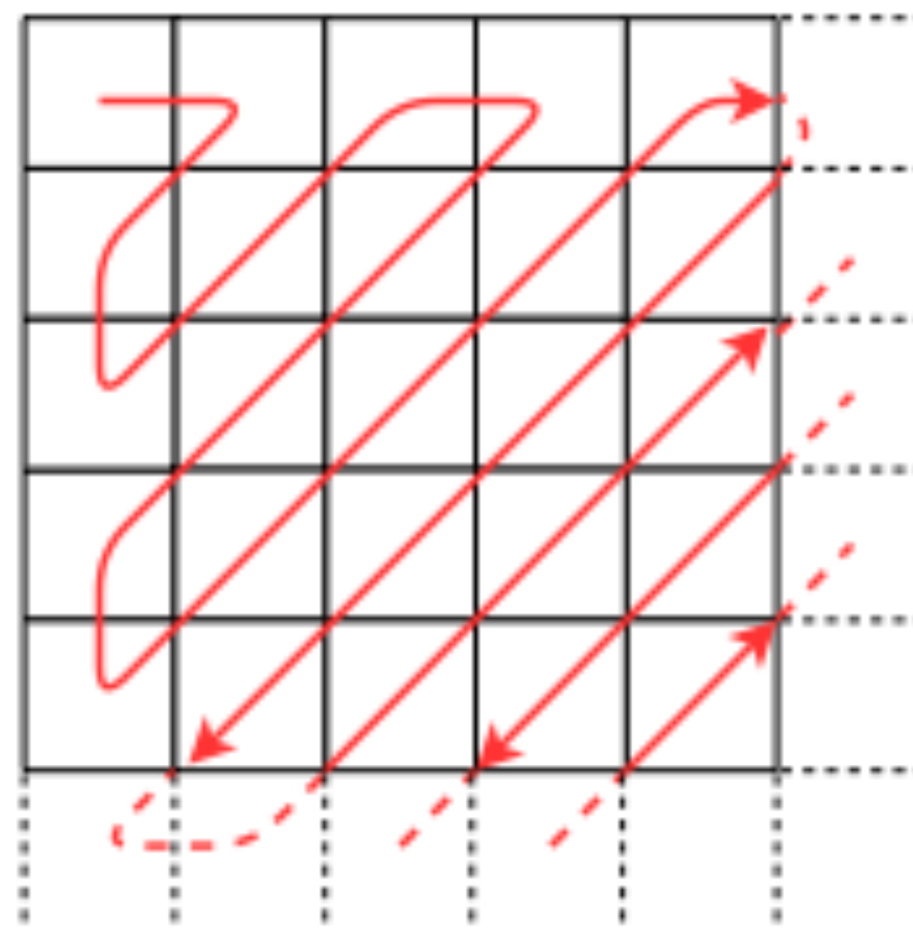
The Rational Numbers

- Suddenly notice counting along finite diagonals
- Able to assign natural number to every rational



Beauty in this Proof

- Aesthetic: not relying on concept of numbers
- Imagination: jump to diagonal path seems spontaneous
- Purposiveness: have successfully ‘caught’ every rational



Cantor Diagonalization

- What about the real numbers?
- Assumption: every real number is included in this list

1.	0	.	1	1	1	1	1
2.	1	.	4	1	4	2	1
3.	3	.	1	4	1	5	9
4.	1	.	7	3	2	0	5
5.	0	.	1	2	5	1	6
6.	2	.	5	7	3	3	8

Cantor Diagonalization

- Notice diagonal entries
- Take number x, consisting of these digits: x=0.44218…
- Choose a number y such that y shares no digits with x, e.g. y=1.73602…
- Conclusion: y not in list i.e. the real numbers are uncountable

1.	**0**	.	1	1	1	1	1
2.	1	.	**4**	1	4	2	1
3.	3	.	1	**4**	1	5	9
4.	1	.	7	3	**2**	0	5
5.	0	.	1	2	5	**1**	6
6.	2	.	5	7	3	3	**8**

Sublimity in this Proof

- **Imagination has failed** to delimit the rational numbers - pain
- **Aesthetic:** only using ‘same’/‘not same’, not properties of numbers
- **Reason stops process** by creating new principle that some infinitely large sets are bigger than others

Key Discoveries

- Imagination is stuck in an iterative loop
- Cantor diagonalization used in Gödel's Theorem, the Halting Problem
- Halting Problem relates to aesthetic process

Morality?

- Kant: sublime experience makes you aware of your moral purpose
- Mathematically sublime proofs may not be moral in the typical sense
- Mathematician may act ‘morally’ by supplying a sublime proof [See: Cheng]

Conclusion:

- Some mathematical proofs can provoke sublime experiences
- These experiences are aesthetically grounded in the Kantian sense
- Problem of accommodating morality

Questions?

Thank you for listening

Sources

- Image: http://www.math.vanderbilt.edu/~schectex/courses/thereals/potential.html
- A. Breitenbach: *Beauty in Proofs*
- W. P. Thurston: *On Proof and Progress in Mathematics*
- E. Cheng: *Mathematics, morally*

Further Reading

- J. W. Dauben: *Georg Cantor, His Mathematics and Philosophy of the Infinite*
- R. Goldstein: *Incompleteness, The Proof and Paradox of Kurt Gödel*
- W. Byers: *How Mathematicians Think*